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On the free field realization of the osp(1|2) current algebra

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ABSTRACT

The free field representation of the osp(1|2) current algebra is analyzed. The four point conformal blocks of the theory are studied. The structure constants for the product of an arbitrary primary operator and a primary field that transforms according to the fundamental representation of osp(1|2) are explicitly calculated.

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The free field constructions are a powerful tool in conformal field theory (CFT). The Feigin-Fuchs formalism [1] has allowed to study the conformal block structure of the minimal models and to obtain the operator algebra of their primary fields [2]. The extension of the Feigin-Fuchs approach to the minimal superconformal models has been performed in refs. [3, 4, 5]. In the case of current algebras the Wakimoto representation [6, 7, 8] plays a similar rôle. Indeed in ref. [9] Dotsenko has presented an analysis of the free field representation of the $SU(2)$ CFT which leads to a direct evaluation of the correlation functions and structure constants of the model.

It is the purpose of this paper to perform a similar study for the case of a CFT enjoying an affine symmetry based on the Lie superalgebra osp(1|2) [10]. To motivate our work let us mention that the osp(1|2) theory is related to the $N = 1$ superconformal minimal models via the quantum hamiltonian reduction [11]. Moreover, the topological version of the osp(1|2) CFT, *i.e.* the osp(1|2)/osp(1|2) coset model, is related to the non-critical Ramond-Neveu-Schwarz superstring [12, 13]. We have undertaken this study with the hope that a complete solution of the osp(1|2) theory could make more explicit these connections.

The osp(1|2) current algebra is generated by three bosonic currents $J_\pm$ and H, together with two fermionic currents $j_\pm$. This algebra can be realized [11] in terms of one scalar field ϕ, two conjugate bosonic fields (w,χ) and one fermionic bc system (denoted by $(\bar{\psi},\psi)$). The dimensions of these fields are $\Delta(w) = \Delta(\bar{\psi}) = 1$ and $\Delta(\chi) = \Delta(\psi) = 0$. The basic operator product expansions (OPE's) will be taken as:

$$w(z_1)\,\chi(z_2) \;=\; \psi(z_1)\,\bar{\psi}(z_2) \;=\; \frac{1}{z_1 - z_2} \qquad\qquad \phi(z_1)\,\phi(z_2) \;=\; -\log\,(z_1 - z_2)\,. \quad (1)$$

In terms of these fields the osp(1|2) currents can be represented as:

$$
\begin{aligned}
J_+ &= w \\
J_- &= -\, w\chi^2 \,+\, i\sqrt{2k+3}\;\chi\partial\phi \,-\, \chi\psi\bar{\psi} \,+ k\partial\chi \,+\, (k+1)\psi\partial\psi \\
H &= -\, w\chi + \frac{i}{2}\sqrt{2k+3}\;\partial\phi \,-\, \frac{1}{2}\psi\bar{\psi} \\
j_+ &= \bar{\psi} \,+\, w\psi \\
j_- &= -\,\chi(\bar{\psi} \,+\, w\psi) \,+ i\sqrt{2k+3}\;\psi\partial\phi \,+\, (2k+1)\partial\psi\,.
\end{aligned}
\tag{2}
$$

In eq. (2) the c-number k is the level of the affine osp(1|2) superalgebra. In this paper we shall restrict ourselves to the case in which k is a positive integer. Recall that k parametrizes the central extension of the algebra. As a CFT, the osp(1|2) current algebra is characterized by its energy-momentum tensor T. The explicit form of T in terms of the currents can be obtained from the Sugawara construction. Using the representation (2) one can easily get T as a function of the free fields:

$$
T \,=\, w\partial\chi \,-\, \bar{\psi}\partial\psi \,-\, \frac{1}{2}\,(\partial\phi)^2 \,+\, \frac{i}{2}\,\alpha_0\,\partial^2\phi\,. \tag{3}
$$

In (3) α_0 is a background charge, whose expression in terms of the level k is given by:

$$
\alpha_0 \,=\, -\frac{1}{\sqrt{2k+3}}\,. \tag{4}
$$

It is easy to verify from equations (3) and (4) that T satisfies the Virasoro algebra with central charge $c = \frac{2k}{2k+3}$.

The irreducible representations of the osp(1|2) superalgebra have been studied in ref. [14]. They are characterized by their isospin j, which can be either integer or half-integer, and the statistics of their highest weight states. We shall denote a general state of the isospin j multiplet by $|j,m>$, where m is the eigenvalue of the generator H ($m = -j, -j+\frac{1}{2}, \cdots, j-\frac{1}{2}, j$). Obviously $|j,j>$ is the highest weight state and the dimensionality of the representation is $4j+1$. If $|j,j>$ is bosonic (fermionic) we will say that the representation is even (odd). Notice that when $j-m$ is integer (half-integer) the states $|j,m>$ and $|j,j>$ have the same (opposite) statistics. In the affine theory we have a primary field Φ^j_m associated to

every state $|j, m>$. It is not difficult to obtain a representation of Φ^j_m within our free field realization. One can check that the operators

$$\Phi^j_m \;=\; \begin{cases} \chi^{j-m}\, e^{-2ij\alpha_0\,\phi} & \text{if } j-m \in \mathbb{Z} \\ \\ \chi^{j-m-\frac{1}{2}}\,\psi\, e^{-2ij\alpha_0\,\phi} & \text{if } j-m \in \mathbb{Z}\, +\frac{1}{2}\,, \end{cases} \tag{5}$$

satisfy all the requirements. First of all it is easy to check that they are primary operators with a conformal weight which is independent of m and given by:

$$\Delta_j \;=\; \frac{j(2j+1)}{2k+3}\,. \tag{6}$$

Moreover, the quantity m is precisely the eigenvalue of the generator H, as it is evident from the OPE

$$H(z_1)\,\Phi^j_m(z_2) \;=\; m\;\frac{\Phi^j_m(z_2)}{z_1 - z_2}\,. \tag{7}$$

Acting with the $J_\pm$ operators we change the value of m by one unit

$$J_\pm(z_1)\,\Phi^j_m(z_2) \;=\; \begin{cases} (j\mp m)\;\frac{\Phi^j_{m\pm1}(z_2)}{z_1-z_2} & \text{if } j-m\in\mathbb{Z} \\ \\ (j\mp m\,-\frac{1}{2})\;\frac{\Phi^j_{m\pm1}(z_2)}{z_1-z_2} & \text{if } j-m\in\mathbb{Z}\,+\frac{1}{2}\,, \end{cases} \tag{8}$$

whereas the fermionic currents $j_\pm$ generate a shift of m by one-half unit

$$j_\pm(z_1)\,\Phi^j_m(z_2) \;=\; \begin{cases} (j\mp m)\;\frac{\Phi^j_{m\pm1/2}(z_2)}{z_1-z_2} & \text{if } j-m\in\mathbb{Z} \\ \\ \pm\;\frac{\Phi^j_{m\pm1/2}(z_2)}{z_1-z_2} & \text{if } j-m\in\mathbb{Z}\,+\frac{1}{2}\,. \end{cases} \tag{9}$$

The screening operators are a basic ingredient in any free field representation.

These are operators that (anti)commute with all the currents. They can be obtained by integrating a local operator over a closed contour:

$$Q = \oint dz\, S(z)\,, \tag{10}$$

where $S(z)$ has conformal weight equal to one and is such that its OPE's with the osp(1|2) currents have only total derivatives. This last condition guarantees the (anti)commutation of Q with $J_{\pm}$, H and $j_{\pm}$. It is easy to verify[11] that S can be taken as:

$$S = (\,\bar{\psi} - w\psi\,)\, e^{i\alpha_0\phi}\,. \tag{11}$$

In order to get integral representations for the conformal blocks of the theory one needs to construct conjugate representations for the primary fields. Let us proceed as it was done in ref. [9] for the $SU(2)$ case. First of all, we ask ourselves which is the operator conjugate to the identity. This operator $\tilde{I}$ must have conformal dimension $\Delta = 0$ and, after modding out spurious states, its OPE's with the currents must only contain regular terms. These conditions allow to determine the form of $\tilde{I}$. One gets:

$$\tilde{I} = w^s\, e^{2is\alpha_0\phi}\,, \tag{12}$$

where $s = -k-1$. The form of $\tilde{I}$ given in equation (12) fixes the charge asymmetry conditions that one must impose to any correlator in order to get a non-vanishing result. Let us suppose that we are computing the vacuum expectation value $< \prod_i O_i >$ where O_i are general operators of the form $O_i = w^{n_i}\,\chi^{m_i}\,e^{i\alpha_i\phi}$. Calling $N(w) = \sum_i n_i$ and $N(\chi) = \sum_i m_i$, one gets after inspecting eq. (12), the following conditions:

$$\begin{aligned} N(w) - N(\chi) &= s \\ \sum_i \alpha_i &= 2\alpha_0 s\,. \end{aligned} \tag{13}$$

As it happened for the $SU(2)$ case (see [9]) it is simple to obtain the expression of

the operator $\tilde{\Phi}^j_j$ conjugate to the highest weight primary field. The result is:

$$\tilde{\Phi}^j_j \;=\; w^{2j+s}\; e^{2i(j+s)\alpha_0\,\phi}\,. \tag{14}$$

Acting on $\tilde{\Phi}^j_j$ with j_- and J_- one gets the explicit form of the remaining operators $\tilde{\Phi}^m_j$ of the conjugate multiplet. The expressions obtained in this way become increasingly involved as m is decreased. To illustrate this fact, let us write the simplest of those fields:

$$\tilde{\Phi}^j_{j-1/2} \;=\; \frac{1}{2j}\,[\,(2j+s)\,\bar{\psi}\,w^{2j+s-1}\;-\;s\,w^{2j+s}\,\psi\,]\,e^{2i(j+s)\,\alpha_0\phi}\,. \tag{15}$$

We shall try now to apply the general formalism developed to the study of the four point conformal blocks of the model. By using the $SL(2,\mathbb{Z})$ projective invariance of the Virasoro algebra one can fix the positions of the four fields involved in the correlator to the values $z_1 \;=\; 0$, $z_2 \;=\; z$, $z_3 \;=\; 1$ and $z_4 \;=\; \infty$. In general we will have to study a vacuum expectation value of the form $<\; \Phi^{j_1}_{m_1}(0)\,\Phi^{j_2}_{m_2}(z)\,\Phi^{j_3}_{m_3}(1)\,\tilde{\Phi}^{j_4}_{m_4}(\infty)\,Q^n\;>$, where the four primary fields of the correlator will be taken to correspond to even osp(1|2) representations. A simple counting using the second selection rule in (13) fixes the number of screening operators that one must insert in the correlator to the value $n \;=\; 2\,(\,j_1+j_2+j_3-j_4\,)$. We shall restrict ourselves to the case in which $j_3 \;=\; j_2$ and $j_4 \;=\; j_1$ with $j_1 \geq j_2$. Notice that in this case $n \;=\; 4j_2$, which in particular means that the number of screening operators must be even. Let us restrict further the values of the m's to be $m_1 \;=\; -j_1$, $m_2 \;=\; j_2$, $m_3 \;=\; -j_2$ and $m_4 \;=\; j_1$. It can be seen that the study of this particular case is enough to determine the operator algebra of the model. Accordingly, let us define the quantity:

$$I(z)\;\equiv <\;\Phi^{j_1}_{-j_1}(0)\,\Phi^{j_2}_{j_2}(z)\,\Phi^{j_2}_{-j_2}(1)\,\tilde{\Phi}^{j_1}_{j_1}(\infty)\,Q^{4j_2}\;>\;. \tag{16}$$

Using eqs. (5), (14), (10) and (11) it is straightforward to get the following repre-

sentation for $I(z)$:

$$I(z) \;=\; \prod_{i=1}^{n} \oint_{C_i} dt_i \, \lambda(z, \{t_i\}) \, \eta(\{t_i\}), \tag{17}$$

where C_i are certain integration contours to be specified, the function $\lambda(z, \{t_i\})$ is given by:

$$\lambda(z, \{t_i\}) \;=<\; e^{-2ij_1\alpha_0\,\phi(0)}\, e^{-2ij_2\alpha_0\,\phi(z)}\, e^{-2ij_2\alpha_0\,\phi(1)}\, e^{2i(s+j_1)\alpha_0\,\phi(\infty)}\, e^{i\alpha_0\,\phi(t_1)} \cdots e^{i\alpha_0\,\phi(t_n)} \;>, \tag{18}$$

and $\eta(\{t_i\})$ is:

$$\begin{aligned} \eta(\{t_i\}) \;=& (-1)^{2j_2} \;<\; (\chi(0))^{2j_1}\, (\chi(1))^{2j_2}\, (w(\infty))^{2j_1+s}\, w(t_1) \,\cdots\, w(t_{2j_2}) \;> \times \\ & \times \;<\; \psi(t_1)\cdots\psi(t_{2j_2})\, \bar{\psi}(t_{2j_2+1})\cdots\bar{\psi}(t_{4j_2}) \;>\; + \text{permutations}. \end{aligned} \tag{19}$$

In (19) we have taken into account the fact that, according to the first condition in (13), one needs to pick up $2j_2$ w fields from the screening charges. The sum over permutations indicated in eq. (19) includes all the possible ways to choose $2j_2$ w fields from the $4j_2$ screening operators. Notice that a different fermionic correlator is associated to each of these elections.

Let us now specify the contours appearing in (17). We shall use the canonical set of integrations that correspond to the s-channel conformal blocks [2, 9]. We will take the first $n-p+1$ integrals along a path lying in the real axis and joining the points $t = 1$ and $t = \infty$, where $1 \leq p \leq n+1$. On the other hand, the remaining $p-1$ integrals will be taken between the points $t = 0$ and $t = z$. The integrations in the intervals $(1, \infty)$ and $(0, z)$ will be considered as ordered integrations. Therefore, denoting the holomorphic conformal block corresponding to the contours just described by $I_p(z)$, we can write:

$$I_p(z) \;=\; \int_1^{\infty} du_1 \cdots \int_1^{u_{n-p}} du_{n-p+1} \int_0^{z} dv_1 \cdots \int_0^{v_{p-2}} dv_{p-1}\, \lambda_p(z, \{u_i\}, \{v_i\})\, \eta_p(\{u_i\}, \{v_i\}), \tag{20}$$

where we have relabelled the integration variables t_i as $u_i = t_i$ for $i = 1, \cdots, n-$

$p+1$ and $v_i = t_{n-p+1+i}$ for $i = 1, \cdots, p-1$. The quantity $\eta_p(\{u_i\},\{v_i\})$ is the function $\eta(\{t_i\})$ after this relabelling. Similarly one can get the expression of $\lambda_p(z,\{u_i\},\{v_i\})$ from eq. (18). Indeed, by applying Wick theorem to the vacuum expectation value of exponentials displayed in eq. (18), one gets:

$$\begin{aligned}\lambda_p(z,\{u_i\},\{v_i\}) &= z^{8j_1j_2\rho}\,(1-z)^{8j_2^2\rho}\prod_{i=1}^{n-p+1} u_i^a\,(u_i-z)^b\,(u_i-1)^b\prod_{i<j}(u_i-u_j)^{2\rho}\times \\ &\times\prod_{i=1}^{p-1} v_i^a\,(z-v_i)^b\,(1-v_i)^b\prod_{i<j}(v_i-v_j)^{2\rho}\prod_{i=1}^{n-p+1}\prod_{j=1}^{p-1}(u_i-v_j)^{2\rho},\end{aligned} \tag{21}$$

where we have defined

$$\rho = \alpha_0^2/2 = \frac{1}{2(2k+3)}\,, \qquad a = -2j_1\alpha_0^2\,, \qquad b = -2j_2\alpha_0^2\,. \tag{22}$$

The physical correlation function $G(z\,,\,\bar{z})$ is obtained by combining, in a monodromy invariant form, holomorphic and antiholomorphic blocks:

$$G(z\,,\,\bar{z}) = \sum_p X_p\,|\,I_p(z)\,|^2\,, \tag{23}$$

where the constants X_p will be specified later.

A simple analysis of the representation given in eq. (20) allows to determine the non-analytic behaviour of the functions $I_p(z)$ around the point $z=0$. The result is:

$$I_p(z) \sim N_p\, z^{\Delta_r - \Delta_{j_1} - \Delta_{j_2}}\,, \tag{24}$$

where N_p is a constant and the exponent Δ_r is the conformal weight of a primary field of isospin $r = j_1 + j_2 + \frac{1-p}{2}$. This number r has the interpretation of the isospin of the s-channel intermediate state. In order to confirm this identification, let us notice that as $p = 1, \cdots, 4j_2+1$ the values taken by r are $j_1 - j_2$, $j_1 - j_2 + \frac{1}{2}, \cdots,\ j_1 + j_2$. Remarkably these are the isospin values that are obtained when one

performs the tensor product of two osp(1|2) irreducible representations of isospins j_1 and j_2 [14]. It is interesting to compare this result with the $SU(2)$ case in which only isospins $j_1 - j_2$, $j_1 - j_2 + 1, \cdots, j_1 + j_2$ are obtained.

The coefficients N_p in (24) are given by $4j_2$-dimensional integrals which are, in general, quite hard to evaluate. For this reason let us consider from now on the case $j_2 = 1/2$. If we simply put $j_1 = j$, we have in this case three two-dimensional integrals to compute:

$$N_1(j) = -\int_1^\infty du_1 \int_1^{u_1} du_2 \,(u_1u_2)^{a+b}\,(u_1-1)^b\,(u_2-1)^b\,(u_1 - u_2)^{2\rho-1} \times$$

$$\times \left[\frac{2j}{u_1} + \frac{1}{u_1-1} + (u_1 \leftrightarrow u_2) \right]$$

$$N_2(j) = -\,2j \int_1^\infty du_1\, u_1^{a+b+2\rho-1}(u_1-1)^b \int_0^1 dv_1\, v_1^{a-1}\,(1-v_1)^b$$

$$N_3(j) = -\,2j \int_0^1 dv_1 \int_0^{v_1} dv_2\, v_1^a\, v_2^a\,(1-v_1)^b\,(1-v_2)^b\,(v_1-v_2)^{2\rho-1} \left[\frac{1}{v_1} + (v_1 \leftrightarrow v_2)\right]. \tag{25}$$

The integral for $N_2(j)$ is easy to calculate since it factorizes into two one dimensional integrals which are given in terms of the Euler Γ-functions. The result is:

$$N_2(j) = -2j\, \frac{\Gamma(-a-2b-2\rho)\Gamma(1+b)}{\Gamma(-a-b-2\rho+1)}\, \frac{\Gamma(a)\Gamma(1+b)}{\Gamma(1+a+b)}\,. \tag{26}$$

Moreover, the quantities $N_1(j)$ and $N_3(j)$ can be put in terms of the basic integrals of the Selberg type:

$$J(\alpha,\beta,\gamma) \equiv \int_0^1 du_1 \int_0^{u_1} du_2 \,[\, u_1^{\alpha+1}u_2^{\alpha} + (u_1 \leftrightarrow u_2)\,]\,(1-u_1)^\beta(1-u_2)^\beta(u_1 - u_2)^{2\gamma}\,. \tag{27}$$

In fact, after some simple manipulations one can write:

$$
\begin{aligned}
N_1(j) &= J(-1-a-2b-2\rho, b, \rho - \frac{1}{2}) \\
N_3(j) &= -\, 2j J(a-1, b, \rho - \frac{1}{2})\,.
\end{aligned}
\tag{28}
$$

The function $J(\alpha, \beta, \gamma)$ is easily obtained from the integrals evaluated in ref. [2]. One finds:

$$
J(\alpha, \beta, \gamma) \;=\; 2\,\frac{\Gamma(2\gamma)}{\Gamma(\gamma)}\,\frac{\Gamma(1+\alpha)\Gamma(1+\beta)}{\Gamma(2+\alpha+\beta+\gamma)}\,\frac{\Gamma(2+\alpha+\gamma)\Gamma(1+\beta+\gamma)}{\Gamma(3+\alpha+\beta+2\gamma)}\,. \tag{29}
$$

Let us now see how one can get the operator algebra of the theory from our results. First of all, it is clear from eqs. (23) and (24) that the coefficients appearing in the operator algebra must be related to the constants:

$$
S_p(j) \;=\; X_p\,(N_p(j))^2\,. \tag{30}
$$

Indeed $S_p(j)$ is the coefficient of the non-analytic singularity corresponding to the channel p in the physical correlator $G(z\,,\,\bar{z})$. Moreover the general form of the constants X_p has been obtained in ref. [2] from the integral representation of the blocks. Adapting this result to our case, we can write:

$$
\begin{aligned}
X_p \;=& \prod_{i=1}^{p-1} s(i(\rho-\frac{1}{2}))\prod_{i=0}^{p-2}\frac{s(a+i(\rho-\frac{1}{2}))\,s(1+b+i(\rho-\frac{1}{2}))}{s(1+a+b+(p-2+i)(\rho-\frac{1}{2}))}\,\times \\
&\times\prod_{i=1}^{3-p} s(i(\rho-\frac{1}{2}))\prod_{i=0}^{2-p}\frac{s(1-a-2b-2\rho+i(\rho-\frac{1}{2}))\,s(1+b+i(\rho-\frac{1}{2}))}{s(1-a-b+(i-p)(\rho-\frac{1}{2}))},
\end{aligned}
\tag{31}
$$

where $s(x) \equiv \sin(\pi x)$.

The operator algebra of the theory has the general form:

$$\Phi^{j_1}_{m_1}(z_1,\bar z_1)\,\Phi^{j_2}_{m_2}(z_2,\bar z_2)\;=\;\sum_{r,m} D^{r,m}_{j_1,m_1;j_2,m_2}\;\Big[\,\frac{\Phi^{r}_{m}(z_2,\bar z_2)}{|z_1\,-\,z_2|^{2(\Delta_{j_1}+\Delta_{j_2}-\Delta_r)}}+O(z_1-z_2)\Big].\tag{32}$$

In order to extract the structure constants $D^{r,m}_{j_1,m_1;j_2,m_2}$ of the theory from the quantities $S_p(j)$ one must conveniently normalize the later. To fix this normalization let us recall that the two-point functions of the model must be normalized as:

$$<\;\Phi^{j_1}_{m_1}(z_1,\bar z_1)\,\Phi^{j_2}_{m_2}(z_2,\bar z_2)\;>=\frac{\delta_{j_1,j_2}\,\delta_{m_1,-m_2}}{|z_1\,-\,z_2|^{4\Delta_{j_1}}}\,,\tag{33}$$

which implies the following constraint for the structure constants:

$$D^{0,0}_{j_1,m_1;j_1,-m_1}\;=\;1\,.\tag{34}$$

Let us now see how one can implement this constraint in our formalism. The condition (34) is automatically fulfilled if we divide the coefficients $S_p(j)$ by the one that corresponds to the unit operator in the intermediate state. Clearly the s-channel isospin r is equal to zero only when $j=1/2$ and $p=3$(recall that $j\geq 1/2$). Applying this reasoning to the $r=j\pm\frac{1}{2}$ cases, one is led to write:

$$\Big[\,D^{j-\frac12,j-\frac12}_{j,j;\frac12,-\frac12}\,\Big]^2\;=\;\frac{S_3(j)}{S_3(\frac12)}\,,\qquad\qquad \Big[\,D^{j+\frac12,j-\frac12}_{j,j;\frac12,-\frac12}\,\Big]^2\;=\;\frac{S_1(j)}{S_3(\frac12)}\,.\tag{35}$$

Using eqs. (26), (28), (29) and (31) and the relation $s(x)\,\Gamma^2(x)\;=\;\pi\Gamma(x)/\Gamma(1-x)$, the structure constants of eq. (35) are found to be:

$$\Big[\,D^{j-\frac12,m_1+m_2}_{j,m_1;\frac12,m_2}\,\Big]^2\;=\;\Big[\,C^{j-\frac12,m_1+m_2}_{j,m_1;\frac12,m_2}\,\Big]^4\;\frac{\Gamma(\frac12\,+\,\rho)\,\Gamma(\frac12\,-\,3\rho)}{\Gamma(\frac12\,-\,\rho)\,\Gamma(\frac12\,+\,3\rho)}\,\frac{\Gamma(\frac12\,+\,(4j+1)\rho)\,\Gamma(\frac12\,-\,(4j-1)\rho)}{\Gamma(\frac12\,-\,(4j+1)\rho)\,\Gamma(\frac12\,+\,(4j-1)\rho)}$$

$$\Big[\,D^{j+\frac12,m_1+m_2}_{j,m_1;\frac12,m_2}\,\Big]^2\;=\;\Big[\,C^{j+\frac12,m_1+m_2}_{j,m_1;\frac12,m_2}\,\Big]^4\;\frac{\Gamma(\frac12\,+\,\rho)\,\Gamma(\frac12\,-\,3\rho)}{\Gamma(\frac12\,-\,\rho)\,\Gamma(\frac12\,+\,3\rho)}\,\frac{\Gamma(\frac12\,+\,(4j+3)\rho)\,\Gamma(\frac12\,-\,(4j+1)\rho)}{\Gamma(\frac12\,-\,(4j+3)\rho)\,\Gamma(\frac12\,+\,(4j+1)\rho)},\tag{36}$$

where $C^{r,m_1+m_2}_{j,m_1;\frac12,m_2}$ are the osp(1|2) Clebsch-Gordan coefficients. These coefficients

have been obtained in ref. [14]. Actually only the values:

$$\begin{aligned} C^{j-\frac{1}{2},j-\frac{1}{2}}_{j,j;\frac{1}{2},-\frac{1}{2}} &= -\,1 \\ C^{j+\frac{1}{2},j-\frac{1}{2}}_{j,j;\frac{1}{2},-\frac{1}{2}} &= -\,C^{j,j-\frac{1}{2}}_{j,j;\frac{1}{2},-\frac{1}{2}} = \frac{1}{\sqrt{2j+1}}\,, \end{aligned} \tag{37}$$

are needed in our present calculation.

It is interesting to point out that the state $|j\,,j-\frac{1}{2}\,>$ appearing in the tensor product $|j\,,j\,\,>\otimes|\frac{1}{2}\,,-\frac{1}{2}\,>$ has negative norm and has to be normalized to -1. In fact this $|j\,,j-\frac{1}{2}\,>$ state has bosonic statistics which means that it belongs to an odd representation, despite of the fact that we are only considering products of even representations. These considerations naturally lead us to define

$$\Big[\,D^{j,j-\frac{1}{2}}_{j,j;\frac{1}{2},-\frac{1}{2}}\,\Big]^2 = -\frac{S_2(j)}{S_3(\frac{1}{2})}\,. \tag{38}$$

Again it is a simple exercise to work out the right-hand side of (38) and arrive at the following expression for the structure constants of this channel:

$$\begin{aligned} \Big[\,D^{j,m_1+m_2}_{j,m_1;\frac{1}{2},m_2}\,\Big]^2 = \Big[\,C^{j,m_1+m_2}_{j,m_1;\frac{1}{2},m_2}\,\Big]^4\,2^{4-8\rho}\,\rho^2\,(2j)^2\,(2j+1)^2\,\,\frac{\Gamma(\frac{1}{2}\,+\,\rho)\,\Gamma(\frac{1}{2}\,-\,3\rho)}{\Gamma(\frac{1}{2}\,-\,\rho)\,\Gamma(\frac{1}{2}\,+\,3\rho)}\times \\ \times\Big[\,\frac{\Gamma(1\,-\,\rho)}{\Gamma(\rho)}\,\Big]^2\,\Big[\,\frac{\Gamma((4j+2)\rho)\,\Gamma(-4j\rho)}{\Gamma(1\,-\,(4j+2)\rho)\,\Gamma(1+\,4j\rho)}\,\Big]^2, \end{aligned} \tag{39}$$

where we have used the duplication formula for the Γ-function, $\sqrt{\pi}\,\Gamma(2z) = 2^{2z-1}\Gamma(z)\,\Gamma(z+\frac{1}{2})$, in order to arrive at the final result. It is easy to verify that the right-hand side of eq. (39) is non-negative, which confirms the correctness of our prescription (38).

Notice that we have only computed the structure constants of eqs. (36) and (39) for a particular case of m_1 and m_2. The operator algebra constants for different values of these numbers can be obtained by studying

some other correlators. One can, for example, analyze the vacuum expectation values $< \Phi^{j}_{-j}(0)\,\Phi^{\frac{1}{2}}_{-m}(z)\,\Phi^{\frac{1}{2}}_{m}(1)\,\tilde{\Phi}^{j}_{j}(\infty)\,Q^{n} >$ for $m = 0, 1/2$ and $< \Phi^{j}_{-j+\frac{1}{2}}(0)\,\Phi^{\frac{1}{2}}_{-m}(z)\,\Phi^{\frac{1}{2}}_{m}(1)\,\tilde{\Phi}^{j}_{j-\frac{1}{2}}(\infty)\,Q^{n} >$ for $m = 0, \pm 1/2$. In this last case the explicit form of the the conjugate operator $\tilde{\Phi}^{j}_{j-1/2}$ (see eq. (15)) will be needed. These calculations can be done by the same method we have followed for the four point function (16). The results for the structure constants coincide with eqs. (36) and (39).

Summarizing, we have been able to study the general structure of the four point functions of the osp(1|2) current algebra by means of a free field representation. The formalism allows to obtain the operator algebra of the model, as we have illustrated in the particular case in which one of the primary fields that are multiplied carries the quantum numbers of the fundamental representation. Much work remains to be done. First of all, it would be interesting to obtain the operator algebra for the case of two arbitrary isospins. In order to achieve this purpose one must tackle the combinatorics and the multiple integrals that one has to deal with in this general case. Another interesting problem is the study of the quantum hamiltonian reduction in this model. We hope that our results might help to shed light on the relation between the osp(1|2) current algebras and the minimal superconformal models at the level of correlators.

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REFERENCES

1. B. L. Feigin and D. B. Fuchs,, *Funct. Anal. and Appl.* **13, No.4**(1979), 91, *Funct. Anal. and Appl.* **16, No.2**(1982), 47.
2. Vl.S.Dotsenko and V. A. Fateev, *Nucl. Phys.* **B240**(1984), 312, *Nucl. Phys.* **B251**(1985), 691, *Phys. Lett.* **B154**(1985), 291.
3. H. Eichenherr, *Phys. Lett.* **B151**(1985), 26; M.A. Bershadsky, V. G. Knizhnik and M. G. Teitelman, *Phys. Lett.* **B151**(1985), 31; D. Friedan, Z. Qiu and S. Shenker, *Phys. Lett.* **B151**(1985), 37.
4. Y. Kitazawa et al., *Nucl. Phys.* **B306**(1988), 425.
5. L. Alvarez-Gaume and P. Zaugg, *Ann. Phys.* **215**(1992), 171.
6. M. Wakimoto, *Comm. Math. Phys.* **104**(1986), 604.
7. A. Gerasimov et al., *Int. J. Mod. Phys.* **A5**(1990), 2495.
8. B. L. Feigin and E. V. Frenkel in "Physics and Mathematics of Strings", edited by L. Brink et al., World Scientific, 1990, *Comm. Math. Phys.* **128**(1990), 161, *Lett. Math. Phys.* **19**(1990), 307.
9. Vl. S. Dotsenko, *Nucl. Phys.* **B338**(1990), 747, *Nucl. Phys.* **B358**(1991), 547.
10. For an account of the general theory of Lie superalgebras see M. Scheunert, "The Theory of Lie Superalgebras", *Lect. Notes in Math.* 716, Springer-Verlag, Berlin (1979).
11. M. Bershadsky and H. Ooguri, *Phys. Lett.* **B229**(1989), 374.
12. J. B. Fan and M. Yu, "G/G Gauged Supergroup Valued WZNW Field Theory", Academia Sinica preprint AS-ITP-93-22, hep-th/9304123.
13. I. P. Ennes, J. M. Isidro and A. V. Ramallo, *Int. J. Mod. Phys.* **A11**(1996), 2379.

14. A. Pais and V. Rittenberg, *J. Math. Phys.* **16**(1975), 2063; M. Scheunert, W. Nahn and V. Rittenberg, *J. Math. Phys.* **18**(1977), 155.